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Nenad Smokrović, Majda Trobok
University of Rijeka, Faculty of Humanities and Social Sciences, Sveučilišna avenija 4, HR–51000 Rijeka
nenad@ffri.hr, trobok@ffri.hr

Mathematics and Pragmatic Naturalism

Abstract
In this paper we shall concentrate on the issue of those ways of knowing in mathematics that have traditionally been taken to support apriorism*. We shall do it by critizing pragmatic naturalism in the philosophy of mathematics, and in particular its historical approach in denying any role to* apriority *in mathematical epistemology. The version of pragmatic naturalism we shall be analyzing is Kitcher's. In the paper we shall first set out a brief survey of the relevant features of Kitcher's pragmatic naturalism in the philosophy of mathematics and then indicate the points that provoke our disagreement.*

Key words
pragmatic naturalism, philosophy of mathematics, Platonism, Philip Kitcher

(Pragmatic) naturalism

One way of formulating naturalism is to take it to be the view according to which the entities and processes we admit in our theories ought to be concordant with the findings of disciplined inquiry. On Kitcher's view, it extends beyond the natural sciences to investigations of human life and culture. As Goodman would say, there should not be more things dreamt of in philosophy than there are in heaven and earth.

Distinctive part of naturalistic approach in the philosophy of mathematics consists in rejecting Platonism, which, according to Kitcher,[1] presupposes a direct insight into an objective mathematical realm and implies strong apriority in belief forming processes as well as in their justification, together with infallibilism concerning mathematical results thus obtained.

The pragmatic part of Kitcher's intriguing theory claims that mathematical truth "is what rational inquiry will produce in the long run". Since he also claims that "there is no independent notion of mathematical truth", it is the rationality of the inquiry that dictates what is ultimately to be counted as true. True mathematical statements are those that "in the limit of the development of rational mathematical inquiry, our mathematical practice contains."(Kitcher, 1988)

Accounting for the *rational inquiry in the long run*, and depicting the view that opposes it, Kitcher attacks the "Myth of the synchronic reconstruction of

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As we shall see, those who defend standard Platonism would reject Kitcher's overall characterisation of Platonism.

the knowledge of the lone individual" according to which "it is possible, in principle, to reconstruct an individual's knowledge by tracing the chains of justification *within* that person's beliefs and experiences." (Kitcher, 1998, p. 61)

In "Epistemology without History is Blind", he succinctly formulates his target:

> "The methods to be sought for the extension of knowledge are no longer intended to inform us about the conditions under which an individual – in splendid isolation – *is justified in believing some hypothesis on the basis of some presumptive class of 'evidence statements'*",

and in the same passage he offers an epistemic alternative to the view attacked that highlights the "rational inquiry in the long run":

> "… but rather how subjects, individually and collectively, are justified in modifying a heterogeneous corpus of statements they have inherited from their predecessors." (Kitcher, 2011, p. 510)

Kitcher's formulation of elements of the collective epistemology, his belief in the decisive role of the history and language games "that are worth playing" that amounts to the view of naturalized pragmatic epistemology still allow certain vagueness concerning the notion of mathematical truth. Since, according to Kitcher, there is no independent notion of mathematical truth, what are mathematical statements about?

Kitcher is clear about mathematical statements claiming that they are about our abilities to manipulate actual worldly, and possibly, imagined objects. If mathematics is about our practice and our manipulation abilities, how does the objectivity of mathematics enter the picture?

Objections to Kitcher's pragmatic naturalism

Our objections will address each of the elements of the theory we mentioned. We shall firstly concentrate on the way Kitcher rejects the Platonist approach, secondly we shall focus on the plausibility of the justification proposal in the frame of "dynamic picture", and thirdly on the objectivity of mathematics.

Let us start by taking a critical look at some of the reasons Kitcher offers for rejecting Platonism.[2] If one postulates a Platonic domain of mathematical objects, the epistemological problem of how we know anything about mathematics seems to be particularly acute since mathematics (on the face of it) does not deal with physical entities but rather with abstract and necessary ones. The main epistemological problem for Platonism was familiar even to Plato, but its most prominent formulation in the contemporary philosophy of mathematics is due to Paul Benacerraf.[3] It can be formulated briefly in the following way: if the causal theory of knowledge is true and mathematical objects are abstract and therefore causally inert, then no mathematical knowledge is possible. The obvious conclusion to be drawn, since we do have some mathematical knowledge, is that Platonism is untenable. One possible answer (that of standard Platonism) is that we do have some sort of causal interaction with abstract mathematical objects. According to standard Platonism, Platonist intuitions allow us to grasp the basic mathematical objects and theorems. In this paper we are going to deal with Benacerraf's dilemma *just* in the context of both Kitcher's methodological route according to which "history is the teacher of epistemology" and his refusal of Platonic intuitions, i.e. Platonic perception. Our point is that to follow Kitcher's methodological route amounts to actually endorsing the existence of Platonist intuitions and hence that defending such a methodological route is not a good strategy if the aim is to deny the existence of Platonist intuitions.

Kitcher is particularly critical of the argument according to which (at least some) mathematical results are self-evident and finds it most unsatisfactory since it implies that "mathematical knowledge can be completely reconstructed for individuals, here and now, without reliance on historical tradition." (Kitcher, 2011, p. 517) As he points out:

> "If particular kinds of propositions appear evident to us, even so evident that we cannot see how they could be wrong, that is simply a result of the power of the tradition in which we stand to give them this appearance….The strong notion of the *a priori* yearns for an independence from tradition that is unattainable." (Kitcher, 2011, p. 511)

Besides that, he notes that the presumed "seeing" of what is "self-evident" clashes with the fact that most mathematical theorems are the results of hard work. What is evident now was not evident to great mathematicians before us and secondly, in such a picture the historical process of struggling to get certain results is not taken into account.

Let us start from the passage just quoted, stressing the role of tradition in mathematics. When talking about the history of mathematics there are however, results that have been achieved due to the independence from, rather than embedment in, the socio-historical-philosophico-mathematical[4] tradition. Talking about the tradition in which we stand, there are revolutions in the history of mathematics in which what was evident to one was not evident to anyone else from the (same) mathematical community. Gödel e.g., underlines that his results were a trivial consequence of his predecessors' achievements but because of their socio-philosophico-mathematical limitations they could not have reached them. In a letter to Hao Wang (letter of 7 December 1967),[5] he writes:

> "… I may add that my objectivistic conception of mathematics and metamathematics in general, and of transfinite reasoning in particular, was fundamental also to my other work in logic".

In another letter to Hao Wang (letter of 7 March 1968), Gödel also suggests that the philosophical views of other mathematicians have influenced their work. He relates that, even though a paper by Skolem written in 1922 contained the core of the proof of the completeness of first order logic as almost a trivial consequence of Skolem's results, Skolem did not draw this conclusion due to his philosophical views:

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Our aim in this section of the paper is not to defend Platonism nor to deal with the problems Platonism has to face. The intention is just to show that Kitcher's methodological route does not *per se* exclude the standard Platonist epistemic route, which is based on Platonist intuitions.

3
In his "Mathematical Truth", *Journal of Philosophy* 70 (1973), pp. 661–679.

4
The expression 'socio-historical-philosophico-mathematical' is used to include the social and historical components as well, apart from the more obvious philosophical and mathematical ones. Many controversies in the history of mathematics were raised due to the socio-philosophical (hence historical) climate of that time. A good example are Cantor's papers, which we mention below, that he could not publish since the editors thought the papers were written "years too soon".

5
See Gödel's letters to Wang published in Wang, 1974, pp. 8–11.

"I am still *perfectly convinced* that reluctance to use non-finitary concepts and arguments in metamathematics was the primary reason why the completeness proof was not given by Skolem or anybody else before my work."

This is not to say that Gödel did not act within certain socio-historical-philosophico-mathematical conditions throughout his education and work. Gödel's theorems were hence the result of his independence from, rather than embedment in, the socio-historical-philosophico-mathematical tradition – he was capable of going beyond anything traditionally accepted in his time, being an "*avant-garde*" mathematician and logician. Similarly with the cases of convergent series discovered by Archimedes, or Cantor's results (some of which rejected for publication because "*... about one hundred years too soon*") or the example of Ramanujan's who was in his early period, rich with results, quite isolated from any official tradition; the living Indian tradition was rudimentary in relation to his far reaching insights. Apart from that, Gödel's thesis that axioms of set theory force themselves upon us as being true does *not* amount to accusing his predecessors of being ignorant or to the claim that the historical path is irrelevant or that knowledge can be "*completely* reconstructed for individuals, [...], without reliance on historical tradition". Quite the contrary, (Gödelian) Platonism respects the historical development of mathematical knowledge. The methodological analogy between mathematics and the natural sciences is one of Platonism's tenets; as Newton was able to see further only because he stood on the shoulders of giants, so was Gödel.

Moreover, as it is the case with visual perception, intuition (or Platonic perception) does not preclude fallibilism. As we can misperceive and hence wrongly infer in the natural sciences investigations, we can "see" false results or have false intuitions in mathematics. Examples are legion. Let us mention the wrong but self-evident assumption that every continuous function is derivable for each value of the domain, which Weierstrass proved to be false.

Furthermore, no one could possibly deny that hard work is involved in any activity such as mathematics, nor it is denied by asserting the existence of mathematical intuitions. Platonism is the view that *at least some* mathematical truths are obtained by platonically perceiving them, while the most part of it is, as Kitcher correctly points out, the result of background knowledge, socio-historical-mathematical interests in certain areas of mathematics, and hard work in finding new results connected with a plethora of possible motivations.

Even though Kitcher rightly underlines that the best strategy is to suppose that the epistemological route follows the historical one, is Dummett not equally right in saying that Platonism could be easily rejected if it did not have followers like Frege or Gödel (and we might add Ramanujan or Hardy or Cantor etc.)? And since mathematical intuitions are not just theoretical presuppositions of the historians but they are asserted by working mathematicians themselves, we might have good reasons to transpose this fact from the history of mathematics, right into the core of our epistemology.

If one thinks, as Kitcher does, that history is the teacher of epistemology, one is certainly not in the position to reject this transposition. Kitcher writes as if the point of Gödel's Platonism is dependent on axioms, rather than theorems, being specifically the object of immediate intuition. However, a Platonist can wholeheartedly agree that axioms are arrived at "through symbolic manipulations, related to and inspired by previously posed problems"; in this case, as Russell was first to see, the manipulation aims at systematizing the accepted simple theorems, and they are in turn often, e.g. prominently in simple arithmetic accepted because of the obviousness. We are thus back at the square one, in need of intuitive resources.[6]

The concentration on the epistemic source of mathematical knowledge would then just shift from the axioms to the results the axioms are based on or are the consequences of. But to shift the attention is not *per se* enough to preclude the existence of mathematical intuitions. But even if we assert (as Kitcher does) that what some mathematicians call "intuition" or even (in the case of Ramanujan) the visitation of the goddess (Namakiri), can be explained as "fine-tuned abilities […] rooted in extant mathematical practice" we face a dilemma. If by 'fine-tuned' Kitcher just means perfect conformity to the extant practice in the profession, is difficult to see how Gödel's non-conformistic example, not to speak about Ramanujan, fits in the picture. If, on the other hand, fine-tuning refers to an impressive ability to reach the truth then it is clearly compatible with the Platonist account on which such a fine-tuning (to the mathematical reality) culminates in intuitive insights.

Let us now focus on the epistemological part of the pragmatic naturalism, more precisely, on its central idea concerning justification of the acquired mathematical beliefs. The core of this appealing and provoking view (the dynamic picture, as Kitcher named it) in nutshell is this: instead of the *static* picture of epistemology depicting a solitary individual justifying his findings according to the "presumptive class of evidence statements", the dynamic picture, fostered by Kitcher is the "one that sees us as dependent on one another and on those who have preceded us and that asks not for the justification of belief but for the justification of change of belief." (Kitcher, 2011, p. 509)

Besides its claim that there are no special cognitive faculties for attaining abstract objects beyond and above ordinary faculties for interacting with the physical world, the new pragmatic epistemology is based on the social and the historical aspects in opposition to the "old" one, based on "the conditions under which an individual – in splendid isolation – *is justified in believing some hypothesis on the basis of some presumptive class of 'evidence statements*'." (Kitcher, 2011, p. 510)

Pointing out that the claim that "mathematicians prove theorems" is not enough, that "proofs must begin somewhere", in the way that mathematicians inherit proven theorems that can be traced back to mathematical axioms and that these theoretical heritages have to be accepted or rejected is a wise methodological advice. Kitcher's idea of history as the "methodologist's laboratory" could be seen as an excellent extension and enhancement of the traditional epistemology. What our objection is targeting is the radical reading of the theory that intends irreconcilably to oppose the view of the old epistemological theory. The major difference is between individual's justification of beliefs, required by the old theory, and the justification of the collectively or individually conceived subjects "in modifying a heterogeneous corpus of statements they have inherited from their predecessors". The radical reading of the claim declares that no presumptive class of "evidence statements" (some kind of necessary conditions for justification) available to the isolated individual is needed for justification, even in the clearest cases. It seems that

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Someone might complaint at this point – and we thank an anonymous referee for this helpful comment – that Kitcher's view might be intended as normative, while we are stressing out exclusively the genetic aspect of it. However, what Kitcher's is endorsing is the view that history *has* to be the teacher of epistemology and if so, Platonist intuitions cannot be ruled out of the mathematical epistemology.

Kitcher, getting rid of the Gödelian direct insight into "objective" mathematical reality, also rejects the picture of the solitary rational decision-maker that, in accordance with some "presumptive class of evidence statements" tries to justify his beliefs and decisions. According to him, it is not an individual, the solitary thinker who has to justify her beliefs. Justification comes partly "horizontally", through the routes of historical heritage, partly "vertically" from the language games played inside a particular scientific community, its approvals and disapprovals. It is beyond any doubt that "the epistemological order of mathematics broadly recapitulates the historical order". But it does not imply that relying on the history and the scientific community explains entirely how mathematical principles "force themselves" upon the individual thinker. It is not the force of the historical route, "the power of the tradition" and community's approval that determines the justification. Rather, it is the force of the mathematical statement itself, recognized by the individual, and being justified for her in accordance with her inner beliefs and experiences. Even if it is true that inherited statements and language games inside community contribute to the justification of mathematical statements to be modified (or accepted), it is still an individual mathematician that should carry on any particular change in view.

Furthermore, every change in view corresponds to one of the simple patterns: either it is so that one changes one's view from being not-determined to believing that p, or from believing that p to believing non-p or from believing that p to being-not-determined. In the first two cases the justification of change results in the justification of the resulting belief so, Kitcher's contrast between the two is hardly as dramatic as a naïve reader might take it to be. Individual thinking and justification might not, and usually does not, preclude fruitful argumentation between opposing views. Kitcher's targeting idea of the solitary thinker (making his views in splendid isolation) is a straw man, a caricature of the real epistemic practice. A mathematician usually acquires her ideas in argumentative confrontation with members of her scientific community, but it does not mean that the role of individual thinking is something strongly opposed to the epistemic practice of mathematical community.

Furthermore, justification for the modification is an individual act performed either internally, as a "perceiving" of reasons for a judgment, or externally, obtained by tracing back the previous reliability rate of the particular way of judging. At the end of the day, it is always an individual thinker, conceived either at the personal or at the sub-personal level, which has to justify her change in belief. Our next objection concerns the objectivity of mathematical truth. Kitcher claims that there is no independent notion of mathematical truth, rejecting in this way Tarskian concept of truth. Instead:

> "Mathematical truths are true in virtue of stipulations which we set down, specifying conditions on the extensions of predicates *which actually are satisfied by nothing at all but are approximately satisfied by operations we perform*." (Kitcher, 1985, p. 110)

He endorses the notion of truth that nicely fits to his pragmatic picture, the *functional use*, "one that takes 'true' statements to be understood as those statements you're trying to produce in a particular language-game – and it is applicable to a broader class of linguistic practices than those that center on description". Clarifying his statement, Kitcher says that "arithmetic describes those structural features of the world in virtue of which we are able to segregate and recombine objects: the operations of segregation and recombination bring about the manifestation of underlying dispositional traits." (Kitcher, 1985, p. 108) On the other hand, he wants to keep a realistic character and objectivistic flavor in his theory. He says:

"… to present my thesis in a way which will bring out its realist character, we might consider arithmetic to be true in virtue not of what *we can do* to the world but rather of what *the world* will let us do *to it*. To coin a Millian phrase, arithmetic is about 'permanent possibilities of manipulation.'" (Kitcher, 1985, p. 108)

The viable way of keeping realistic flavor of the theory, and hence its objectivism, is to claim that operations we perform on objects somehow strongly correspond to ontological structure of the world that is independent of our cognitive ability. On the other hand, the notion of truth Kitcher endorses does not allow mind-independent truth of mathematical statements. Therefore, two tendencies, pragmatic and realist ones, are in obvious tension and require reconciliation. One tendency pushes the theory in the clear conventionalist and fictionalist direction, while the other pulls it toward a kind of objectivist interpretation.

Our point is that reconciliation between the two is not possible. Let us focus on the claim that arithmetic is true in virtue of "what the world will let us do to it". The role of the world is to restrict possibilities of our operations, or, in other words, to constrain the space of modalities. But, how those modalities afforded by the world present itself to the subject? They could be presented in the *epistemic* or in the *non-epistemic* way. The non-epistemic way in which the world restricts any activity, including our basic mathematical operations, is given as a set of empirical facts. In a way, the structure of the world certainly restricts the flowing of a river between the rocks, as crude facts that the river must "obey". Similarly, we can understand the role the world plays claiming that arithmetic is true in virtue of "what the world will let us do to it". The world lets us do what is in accordance with the prevailing facts determining its structure. In this case, mathematics would be about the world and the way the world is would make mathematical statements true. But, if it is so, the whole pragmaticist project seems redundant. If the modalities we are talking about are epistemic, the thinker somehow has to *perceive* them, they have to play some epistemic role in her manipulations. The obvious problem with this solution is that it brings us back to the area Kitcher wants to avoid at any cost. It seems that the reconciliation between pragmaticism and objectivity cannot, after all, be easily obtained.

References

Kitcher, P., 1985, *The Nature of Mathematical Knowledge*, Oxford University Press.

Kitcher, P., 1988, "Mathematical Naturalism", in: W. Aspray, P. Kitcher (eds.), *History and Philosophy of Modern Mathematics*, University of Minnesota Press, Minneapolis.

Kitcher, P., 1998, "Mill, Mathematics, and the Naturalist Tradition", in: J. M. A. Skorupski (ed.), *The Cambridge Companion to Mill*, Cambridge University Press.

Kitcher, P., 2011, "Epistemology without History is Blind', *Erkenntnis* 75 (3), 505–524.

Wang, H., 1974, *From Mathematics to Philosophy*, Routledge & Kegan Paul, London.

Nenad Smokrović, Majda Trobok

Matematika i pragmatički naturalizam

Sažetak

U ovome radu ćemo se koncentrirati na pitanje onih načina spoznaje u matematici za koje se tradicionalno smatralo da podržavaju apriorizam. To ćemo učiniti kritizirajući pragmatički naturalizam u filozofiji matematike, posebice njegov povijesni pristup u negiranju ikakve uloge apriorizma u matematičkoj epistemologiji. Verzija pragmatičkog naturalizma koju ćemo razmatrati je Kitcherova. U radu ćemo prvo iznijeti sažeti pregled relevantnih obilježja Kitcherovog pragmatičkog naturalizma u filozofiji matematike te potom naznačiti točke koje izazivaju naše neslaganje.

Ključne riječi

pragmatički naturalizam, filozofija matematike, platonizam, Philip Kitcher

Nenad Smokrović, Majda Trobok

Mathematik und pragmatischer Naturalismus

Zusammenfassung

Im vorliegenden Paper konzentrieren wir uns auf die Problematik jener Wege der Erkenntnis innerhalb der Mathematik, denen traditionell Unterstützung des Apriorismus *beigemessen wird. Wir verwirklichen dies, indem wir den pragmatischen Naturalismus in der Philosophie der Mathematik kritisieren, namentlich dessen historische Herangehensweise bei der Verneinung jeglicher Rolle der* Apriorität *in der mathematischen Epistemologie. Die Version des pragmatischen Naturalismus, der wir auf den Grund gehen, ist die von Kitcher. In der Abhandlung machen wir erstens einen Streifzug durch die relevanten Wesenszüge von Kitchers pragmatischem Naturalismus in der Philosophie der Mathematik und geben anschließend Anhaltspunkte an, die unseren Dissens hervorrufen.*

Schlüsselwörter

pragmatischer Naturalismus, Philosophie der Mathematik, Platonismus, Philip Kitcher

Nenad Smokrović, Majda Trobok

Mathématiques et naturalisme pragmatique

Résumé

Dans cet article, nous nous concentrerons sur la question des modes de la connaissance en mathématiques qui ont traditionnellement été considérés comme soutenant l'apriorisme. *Nous le ferons en critiquant le naturalisme pragmatique dans la philosophie des mathématiques, notamment son approche historique de la négation de tout rôle de* l'apriorité *dans l'épistémologie mathématique. La version du naturalisme pragmatique que nous analyserons est celle de Philip Kitcher. Dans l'article, nous présenterons d'abord un court examen des aspects pertinents du naturalisme pragmatique de Philip Kitcher dans la philosophie des mathématiques, puis indiquerons les points qui suscitent notre désaccord.*

Mots-clés

naturalisme pragmatique, philosophie des mathématiques, platonisme, Philip Kitcher